2.36. Deduction: Inference Rules

Here we set out the argument patterns which are treated as ‘basic’ in the deductive method. Since we apply these in the same recursive fashion as the construction rules, we likewise speak of these argument forms as “rules”. They will serve as **inference rules**: rules which take certain sentence(s) as input, and infer a conclusion as output – an output that can itself, of course, then serve as further premise ‘input’, to infer a further conclusion, and so on.

We met one inference rule already: **Vel Elim**. This comes in two forms, fitting a common pattern: if one part of a disjunction is denied, the other part follows validly (the order of premises, as always, having no effect on validity).

Vel Elim ($\vee-$)

$$\begin{array}{c} (\bullet \vee \blacktriangle) \\ \sim\bullet \\ \hline \blacktriangle \end{array} \qquad\qquad \begin{array}{c} (\bullet \vee \blacktriangle) \\ \sim\blacktriangle \\ \hline \bullet \end{array}$$

But others are equally simple argument forms, familiar from our first steps in formal logic. We agreed that the following arguments are (boring but) valid.

It’s sunny **and** it’s warm.

∴ It’s sunny.

It’s sunny **and** it’s warm.

∴ It’s warm.

These illustrate the rule of **Wedge Elimination** (“**Wedge Elim**” for short).

Wedge Elim (∧−)

(● ∧ ▲)	(● ∧ ▲)
●	▲

And here the argument works in reverse as well: if we have both halves of a conjunction as premises, we can validly infer the conjunction.

It’s sunny.
It’s warm.
―――――――
∴ It’s sunny **and** it’s warm.

Since we introduce a new conjunction here, this inference rule is called **Wedge Introduction** (“**Wedge Intro**”).

Wedge Intro (∧+)

●	▲
▲	●
(● ∧ ▲)	(● ∧ ▲)

We recognize a sentence with a ‘double negative’ as equivalent to the one without; so we expect either to follow validly from the other.

It did **not fail to** yesterday.
∴ It’s rained yesterday.

It rained yesterday.
∴ It did **not fail to** rain yesterday.

Adding a pair of tildes is called **Tilde Introduction** (“**Tilde Intro**”).

Tilde Intro (~+)

●
~~ ●

Removing a pair of tildes is **Tilde Elimination** (“**Tilde Elim**”).[1]

Tilde Elim (~–)

~~●
●

[1] These two inference rules are often lumped together under the single title “Double Negation” (as was done earlier in 2.28 § 1). But in deductions it is worth separating them, as they serve different purposes – and one is used much more often than the other.

For purposes of completeness and simplicity of deduction, we add two further rules. If we have either half of a disjunction as a premise, we can infer the entire disjunction.

We’re having truffles.
∴ We’re having **either** truffles **or** grog.

We’re having grog.
∴ We’re having **either** truffles **or** grog.

In formal guise this is **Vel Introduction** (“**Vel Intro**”).

Vel Intro (∨+)

$$\frac{\bullet}{(\bullet \vee \blacktriangle)} \qquad \frac{\blacktriangle}{(\bullet \vee \blacktriangle)}$$

Finally we permit the most timid of inferences: whenever a certain sentence is true, that sentence is true.

It’s sunny.
∴ It’s sunny.

This inference rule is **Repetition**.

Repetititon (**R**)

$$\frac{\bullet}{\bullet}$$

Admittedly, neither of these last two inference rules looks particularly *natural*. But for purposes of deduction we don't ask that they be the sort of inference we make in everyday conversation – only that each count as a **valid** argument, however artificial. With Repetition the validity is obvious: whenever a given sentence is true, that sentence is true. But the same holds for Vel Intro: in a situation where we're having cake, the disjunction "We're having either ice cream or cake" can't be false – for that would be a situation where we have neither.

That last point highlights something simple but significant: though nowhere in deductions do we need to appeal to **truth**, it is all the same no coincidence that the inference rules proposed here are valid on the semantic tests. No surprise either: for the deduction method to be reliable, it had better pick out as valid the same arguments approved by the semantic tests. And that is the case: it can be proven (though we will not do so here) that the deductive and semantic methods agree exactly on which arguments are valid.

In closing we stress a further point of similarity between construction rules and inference rules: not only do both exhibit recursive 'recycling' of outputs, but in both cases we can 'mix and match' different rules – taking the output of one as the input of some other. That's how we constructed disjunctions of negations, negations of conjunctions, and all the rest. And the output (conclusion) of one inference rule can likewise serve as the input (premise) of a second – as the following deduction illustrates.

1. $(P \wedge {\sim}Q)$	Premise
2. $(R \vee Q)$	Premise
	Get: $(P \wedge R)$
3. P	1, $\wedge-$
4. $\sim Q$	1, $\wedge-$
5. R	2, 4, $\vee-$
6. $(P \wedge R)$	3, 5, $\wedge+$

The **output** of $\wedge-$ (on line 4) serves as **input** to $\vee-$ (line 5); and that in turn serves as input for $\wedge+$ (line 6). Such combinations allow the deduction procedure to cast its net over an infinite number of valid argument forms – just as the construction rules covered an infinite number of formal sentences.

Inference Rules (Chapter Two)

Disjunction Rules

Vel Elimination (∨–)

$$\frac{\begin{array}{c}(\bullet \vee \blacktriangle)\\ \sim\bullet\end{array}}{\blacktriangle} \qquad \frac{\begin{array}{c}(\bullet \vee \blacktriangle)\\ \sim\blacktriangle\end{array}}{\bullet}$$

Vel Introduction (∨+)

$$\frac{\bullet}{(\bullet \vee \blacktriangle)} \qquad \frac{\blacktriangle}{(\bullet \vee \blacktriangle)}$$

Conjunction Rules

Wedge Elimination (∧–)

$$\frac{(\bullet \wedge \blacktriangle)}{\bullet} \qquad \frac{(\bullet \wedge \blacktriangle)}{\blacktriangle}$$

Wedge Introduction (∧+)

$$\frac{\begin{array}{c}\bullet\\ \blacktriangle\end{array}}{(\bullet \wedge \blacktriangle)} \qquad \frac{\begin{array}{c}\blacktriangle\\ \bullet\end{array}}{(\bullet \wedge \blacktriangle)}$$

Negation Rules

Tilde Elimination (~–)

$$\frac{\sim\sim\bullet}{\bullet}$$

Tilde Introduction (~+)

$$\frac{\bullet}{\sim\sim\bullet}$$

Repetition (R)

$$\frac{\bullet}{\bullet}$$